

The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1 *Version 2.0*

Date: October 1, 2026

Course: EE 313 Evans

Name: _____

- **Exam duration.** The exam is scheduled to last 75 minutes.
- **Materials allowed.** You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks.** Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed.** As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers.** When justifying your answers, reference your source and page number as well as quote the content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab.** No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test.** All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

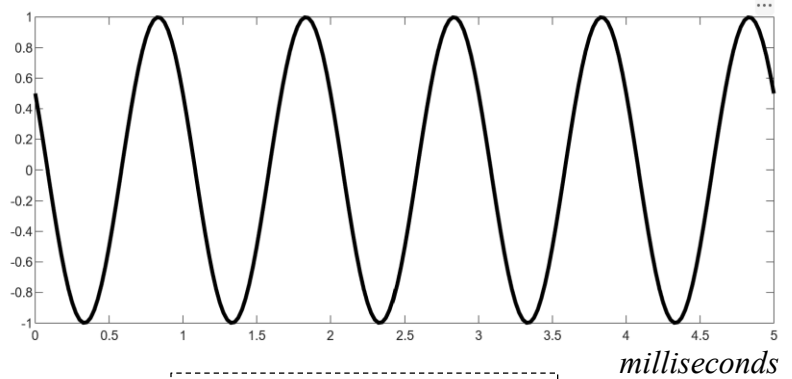
<i>Problem</i>	<i>Point Value</i>	<i>Your score</i>	<i>Topic</i>
1	24		Sinusoidal Signals
2	28		Fourier Series
3	24		Sampling and Aliasing
4	24		Time-Frequency Analysis
<i>Total</i>	100		

Problem 1.1 Sinusoidal Signals. 24 points.

Consider the sinusoidal signal
 $x(t) = A \cos(2\pi f_0 t + \theta)$ with

- amplitude A
- continuous-time frequency f_0 in Hz
- phase θ in radians

On the plot of $x(t)$ on the right, the value of $x(t)$ at $t = 0$ is 0.5. From the plot:



- (a) Estimate the amplitude A . Explain how you estimated the value of this parameter. 6 points.

In a sinusoidal signal, the peak occurs at A and valley occurs at $-A$ because the values of cosine are in the interval $[-1, 1]$. From the plot, $A = 1$.

- (b) Estimate the continuous-time frequency f_0 in Hz. Explain how you estimated the value of this parameter. 6 points.

Approach #1: The duration from one peak to the next is about 1ms and this represents the period T_0 of the sinusoidal signal. The fundamental frequency $f_0 = \frac{1}{T_0} = \frac{1}{0.001s} = 1000 \text{ Hz}$.

Approach #2: From counting the peaks, the plot contains exactly five periods and lasts for 0.005s. Hence, the fundamental period is $T_0 = 0.001s$ and the fundamental frequency is $f_0 = \frac{1}{T_0} = \frac{1}{0.001s} = 1000 \text{ Hz}$.

Approach #3: The plot contains 10 pairs of zero crossings over the 0.005s duration of the signal. Hence, there are 5 periods in 0.005s. The period is $T_0 = \frac{0.005s}{5} = 0.001s$ and the frequency is $f_0 = \frac{1}{T_0} = \frac{5}{0.001s} = 1000 \text{ Hz}$.

- (c) Estimate the phase θ in radians. Explain how you estimated the value of this parameter. 6 points.

Approach #1: Cosine wave is 0.5 at $t = 0$, which means $\cos(\theta) = 0.5$ and hence $\theta = \pm\pi/3$ rad. In this case, it's $\theta = \pi/3$ because the cosine is decreasing in amplitude as t increases.

Approach #2: Estimate the time at the first peak to be about $t_{\text{peak}} = 0.8 \text{ ms}$ and substitute it $x(t_{\text{peak}}) = \cos(2\pi f_0 t_{\text{peak}} + \theta) = 1$ which means $1.6\pi + \theta = 2\pi$ so that $\theta = 0.4\pi$.

- (d) What is the phase of the signal $x(t - 0.001)$? Express the phase you've computed to be in the range $[-\pi, \pi]$. Please show your intermediate steps. 6 points.

$$x(t - 0.001) = A \cos(2\pi f_0 (t - 0.001s) + \theta) = A \cos(2\pi f_0 t - 2\pi f_0 (0.001) + \theta)$$

Hence, the phase is $-2\pi f_0 (0.001s) + \theta = -2\pi (1000 \text{ Hz})(0.001s) + \frac{\pi}{3} = -2\pi + \frac{\pi}{3}$.

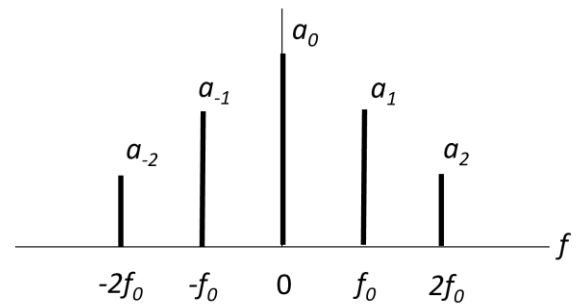
Because phase shifts of integer multiples of 2π have no effect for a sinusoidal signal, we can add 2π to the phase value to obtain $\frac{\pi}{3}$.

Problem 1.2. Fourier Series. 28 points.

Consider a real-valued signal $x(t)$ that is periodic with fundamental frequency f_0 and Fourier series coefficients a_k :

$$a_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j2\pi(kf_0)t} dt$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi(kf_0)t}$$

Spectrum for $x(t)$ 

The spectrum for $x(t)$ is plotted on the right.

Only five Fourier series coefficients a_{-2} , a_{-1} , a_0 , a_1 , a_2 are not zero; all others are zero.

(a) Consider a new signal $y(t) = A x(t)$ where A is a non-zero constant.

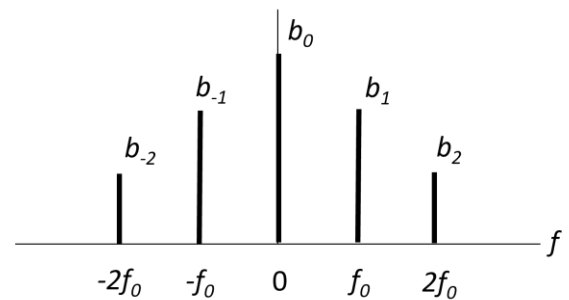
i. What is the fundamental period T_y for $y(t)$ in terms of T_0 ? Why? 3 points.

A continuous-time signal $x(t)$ is periodic with period T_0 if $x(t) = x(t + T_0)$ for $-\infty < t < \infty$. Scaling a signal in amplitude by a constant A does not change the periodicity: $A x(t) = A x(t + T_0)$.

ii. Derive the Fourier series coefficients b_k for $y(t)$ in terms of a_k . 5 points. $y(t) = A x(t) =$

$$A \sum_{k=-\infty}^{\infty} a_k e^{j2\pi(kf_0)t} = \sum_{k=-\infty}^{\infty} A a_k e^{j2\pi(kf_0)t}$$

So, $b_k = A a_k$.

Spectrum for $y(t)$ 

iii. Draw the spectrum for $y(t)$. 6 points. **On right.**

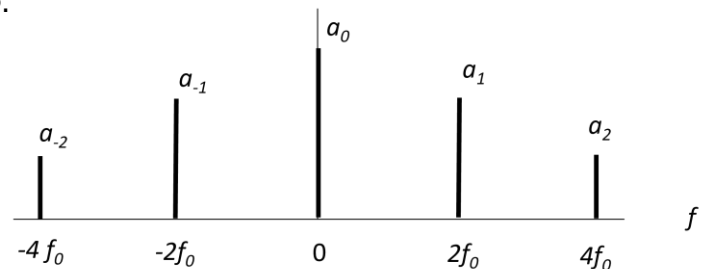
(b) Consider a new signal $v(t) = x(2t)$.

i. What is the fundamental period T_v for $v(t)$ in terms of T_0 ? Why? 3 points.

A continuous-time signal $y(t)$ is periodic with period T_v if $y(t) = y(t + T_v)$ for $-\infty < t < \infty$. Scaling a signal in time changes the period.

One of the periods of $x(t)$ goes from 0 to T_0 which correspond to $y(0) = x(2(0)) = x(0)$ and $y(\frac{1}{2}T_0) = x(2(\frac{1}{2}T_0)) = x(T_0)$.

So, $T_v = \frac{1}{2}T_0$.

Spectrum for $v(t)$ 

ii. Derive the Fourier series coefficients c_k for $v(t)$ in terms of a_k . 5 points.

$$v(t) = x(2t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi(kf_0)(2t)} = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi(k(2f_0))t}$$

So, $c_k = a_k$ but the fundamental frequency for $v(t)$ is $2f_0$.

iii. Draw the spectrum for $v(t)$. 6 points. **See above.**

Problem 1.3. Sampling and Aliasing. 24 points.

Lecture Slides 5-9, 5-12 & 6-7

Homework Prob. 1.1 & 3.2

Consider a continuous-time signal that is a sum of two cosine signals observed for $-\infty < t < \infty$

$$x(t) = \cos(2\pi f_0 t) + \cos(2\pi f_1 t)$$

where $f_1 = 10 f_0$. After sampling at a sampling rate of $f_s > 2 f_0$, the discrete-time signal is

$$x[n] = \cos(\hat{\omega}_0 n) + 1$$

where $\hat{\omega}_0 = 2\pi \frac{f_0}{f_s}$ and frequency f_0 does not alias; however, the frequency f_1 aliases.

What are all the possible values of f_s ? Express your answers for f_s in terms of f_0 .

Note: Problem is analogous to the in-class demo of a videocamera recording a scene using a fixed frame rate (f_s) where the motion of a boat (f_0) is captured correctly but the rotating helicopter blades appear to stand still (f_1): <https://www.youtube.com/watch?v=vr3ngmRuGUc>

Approach #1: After sampling at sampling rate of f_s , continuous-time frequency f_0 does not alias because $f_s > 2 f_0$. However, the continuous-time frequency $f_1 = 10 f_0$ aliases to 0 Hz:

$$x[n] = x(n T_s) = \cos\left(2\pi \frac{f_0}{f_s} n\right) + \cos\left(2\pi \frac{f_1}{f_s} n\right) = \cos(\hat{\omega}_0 n) + 1$$

where $\hat{\omega}_0 = 2\pi \frac{f_0}{f_s}$. After sampling at rate f_s , continuous-time frequency f_1 has replicas $f_1 \pm k f_s$ where k is an integer and its negative counterpart has replicas $-f_1 \pm m f_s$ where m is an integer.

The possible values of f_s for $10 f_0 \pm k f_s = 0$ for $f_s > 2 f_0$ are $\left\{2.5 f_0, \frac{10}{3} f_0, 5 f_0, 10 f_0\right\}$.

Approach #2:

Answer: $x[n] = \cos(\hat{\omega}_0 n) + \cos(\hat{\omega}_1 n)$
 where $\hat{\omega}_1 = 2\pi \frac{f_1}{f_s}$. Since $\cos(\hat{\omega}_1 n) = 1$, $\hat{\omega}_1 \in \{0, 2\pi, 4\pi, \dots\}$.
 So, frequency f_1 has, after sampling, aliased to 0 Hz. This means that $f_s \leq 2 f_1$ which is $f_s \leq 20 f_0$. Since frequency f_0 does not alias, $f_s > 2 f_0$. So, $2 f_0 < f_s \leq 20 f_0$.

Spectrum of $x(t)$

Sampling has the effect in the frequency domain that each frequency component in $x(t)$ is present and also replicates at shifts equal to integer multiples of f_s . We find all four values of f_s that will create a replica of $f_1 = 10 f_0$ to land at the origin. Same for $-f_1$.

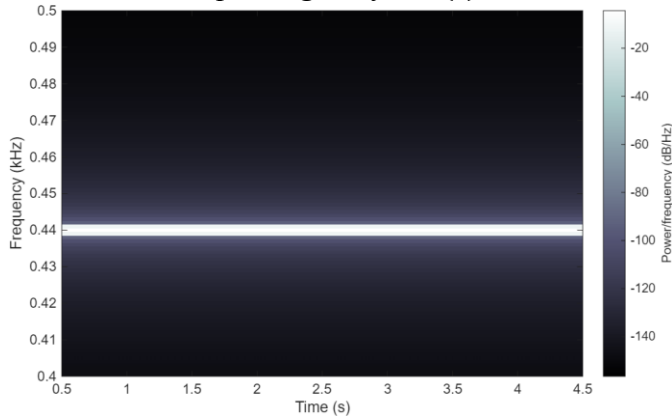
$f_s \in \left\{10 f_0, \frac{10}{2} f_0, \frac{10}{3} f_0, \frac{10}{4} f_0\right\}$

Problem 1.4. Mystery Systems. 24 points.

You're trying to identify unknown systems.

You input a cosine signal $x(t) = \cos(2\pi f_0 t)$ and look at the output to determine what the system is.

Use $f_0 = 440$ Hz. Spectrogram uses sampling rate of $f_s = 8000$ Hz and zooms into frequencies near f_0 .

Spectrogram for $x(t)$ 

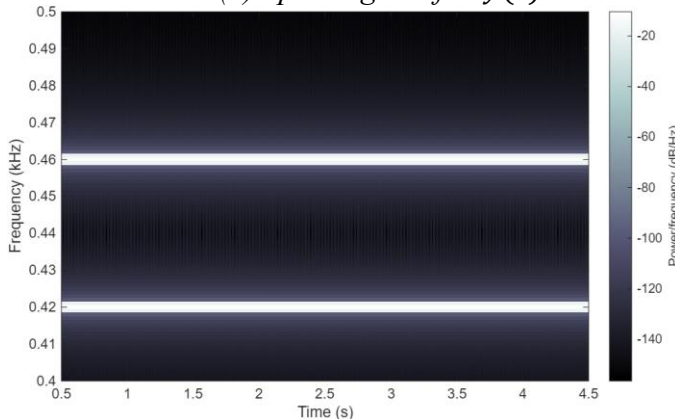
In each part below, identify the unknown system as one of the following **with justification**:

1. squaring block: $y(t) = x^2(t)$
2. cubing block: $y(t) = x^3(t)$
3. amplitude modulation: give frequency f_1 to produce output

$$y(t) = \cos(2\pi f_1 t) x(t)$$
4. frequency modulation: estimate parameters Δf and f_τ to produce output

$$y(t) = \cos\left(2\pi f_0 t + \frac{\Delta f}{f_\tau} \cos(2\pi f_\tau t)\right)$$

(a) A system gives output signal $y(t)$ when the cosine signal $x(t)$ is input. 12 points.

Part (a) Spectrogram for $y(t)$ 

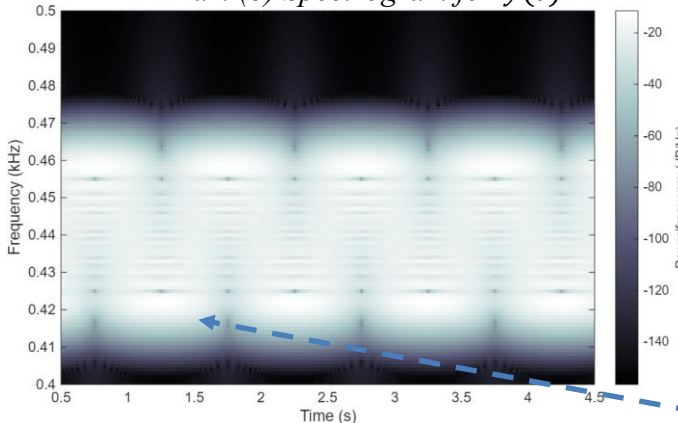
The cosine signal $x(t)$ has a principal frequency at 440 Hz. The spectrogram for $y(t)$ shows two very strong frequency components at 420 Hz and 460 Hz. These frequencies correspond to 440 ± 20 Hz.

This is amplitude modulation with $f_1 = 20$ Hz :

$$y(t) = \cos(2\pi f_1 t) \cos(2\pi f_0 t)$$

$$y(t) = \frac{1}{2} \cos(2\pi(f_0 + f_1)t) + \frac{1}{2} \cos(2\pi(f_0 - f_1)t)$$

(b) Another system gives output signal $y(t)$ when the cosine signal $x(t)$ is input. 12 points.

Part (b) Spectrogram for $y(t)$ 

Not System #1. System #1 would produce frequencies at 0 and 880 Hz which would not be in range of the zoomed in frequency axis.

Not System #2. System #2 would produce two principal frequencies at 440 and 1320 Hz.

Not System #3. System #3 would produce two principal frequencies at $440 \text{ Hz} \pm f_1$.

Yes System #4. The spectrogram has significant frequency content from 420 to 460 Hz, so $\Delta f = 20$ Hz. There is also a thick periodic pattern that looks like $\square$ every second, so $f_\tau = 1$ Hz.

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%% Matlab code to generate the spectrograms for Problem 1.4
fs = 8000;
Ts = 1 / fs;
tmax = 5;
t = 0 : Ts : tmax;

%% Create cosine signal
f0 = 440;
T0 = 1/f0;
x = cos(2*pi*f0*t);

%%% Spectrogram parameters
blockSize = fs;          %%% 1 Hz frequency resolution
overlap = blockSize-1;
%% Plot spectrogram of input signal
figure;
spectrogram(x, blockSize, overlap, blockSize, fs, 'yaxis');
colormap bone;
ylim( [0.4 0.5] );

%% (a) Amplitude modulation
f1 = 20;
y = cos(2*pi*f1*t) .* x;

%% Plot spectrogram of output signal
figure;
spectrogram(y, blockSize, overlap, blockSize, fs, 'yaxis');
colormap bone;
ylim( [0.4 0.5] );

%% (b) Frequency modulation
deltaf = 20;
ftau = 1;
y = cos(2*pi*f0*t + (deltaf/ftau)*cos(2*pi*ftau*t));

%% Plot spectrogram of output signal
figure;
spectrogram(y, blockSize, overlap, blockSize, fs, 'yaxis');
colormap bone;
ylim( [0.4 0.5] );

```